FUNDAMENTAL THEOREM OF CALCULUS

Problems:

1. State "Fundamental Theorem of Calculus".

2. (1) Suppose that f is a continuous function and g is a differentiable function.
   Let
   $$H(x) = \int_a^{g(x)} f(t)\,dt.$$
   Find $H'(x)$.
   (2) Suppose that f is a continuous function and g_1, g_2 are differentiable functions.
   Let
   $$K(x) = \int_{g_1(x)}^{g_2(x)} f(t)dt.$$
   Find $K'(x)$.

3. $f(x) = \begin{cases} x+1 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ x^2 & \text{if } 1 < x. \end{cases}$
   Let
   $$F(x) = \int_{-1}^{x} f(t)dt.$$
   Find $F(x)$ and $F'(x)$.

4. $f(x) = \begin{cases} x+1 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ x^2+1 & \text{if } 1 < x. \end{cases}$
   Let
   $$F(x) = \int_{-1}^{x} f(t)dt.$$
   Find $F(x)$ and $F'(x)$.